HW6, Class 151A, Spring 2012

UCLA

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3.2.10

For $f(-2), f(-1), f(1)$ and $f(2)$ find approximation for $f(0)$ and study sensitivity due to input error.

$x_0 = -2$	$P_0 = f(-2)$			
$x_1 = -1$	$P_1 = f(-1)$	P_{01}		
$x_2 = 1$	$P_2 = f(1)$	P_{12}	P_{012}	
$x_3 = 2$	$P_3 = f(2)$	P_{23}	P_{123}	P_{0123}

$$P_{01}(x) = \frac{(x - x_0)P_1(x) - (x - x_1)P_0(x)}{x_1 - x_0}$$

Hence,

$$\begin{aligned} P_{01}(0) &= 2f(-1) - f(-2) \\ P_{12}(0) &= \frac{1}{2}(f(1) + f(-1)) \\ P_{23}(0) &= 2f(1) - f(2) \end{aligned}$$

We combine and obtain:

$$P_{012}(x) = \frac{(x - x_0)P_{12}(x) - (x - x_2)P_{01}(x)}{x_2 - x_0}$$

$$\begin{aligned} P_{012}(0) &= -\frac{1}{3}(f(-2) - f(1) - 3f(-1)) \\ P_{123}(0) &= -\frac{1}{3}(f(2) - f(-1) - 3f(1)) \end{aligned}$$

combining one more time yields,

$$P_{0123}(0) = \frac{2P_{012}(0) + 2P_{123}(0)}{4} = \frac{1}{6}(f(-2) - 4f(-1) - 4f(1) + f(2))$$

Assuming an input error, $\tilde{f}(-1) := f(-1) + \epsilon_1$ and $\tilde{f}(1) := f(1) + \epsilon_2$, we get the perturbed approx.:

$$\begin{aligned} \tilde{P}_{0123}(0) &= P_{0123}(0) - \frac{2}{3}(\epsilon_1 + \epsilon_2) \\ &=: P_{0123}(0) - d(\epsilon_1, \epsilon_2) \end{aligned}$$

For $\epsilon_1 := 2$ and $\epsilon_2 := -3 : \quad d(\epsilon_1, \epsilon_2) = \frac{2}{3}$.