HW7, Class 151A, Spring 2012
exercises on extrapolation

UCLA

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4.2.8

We have:

$$
\begin{aligned}
f'(x_0) = N_1(h) &= \frac{1}{h}[f(x_0+h) - f(x_0)] - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + \mathcal{O}(h^3) \\
N_1(\frac{h}{2}) &= \frac{2}{h}[f(x_0+\frac{h}{2}) - f(x_0)] - \frac{h}{4}f''(x_0) - \frac{h^2}{24}f'''(x_0) + \mathcal{O}(h^3) \\
N_1(\frac{h}{4}) &= \frac{4}{h}[f(x_0+\frac{h}{4}) - f(x_0)] - \frac{h}{8}f''(x_0) - \frac{h^2}{96}f'''(x_0) + \mathcal{O}(h^3)
\end{aligned}
$$

We eliminate the h term first:

$$
\begin{aligned}
f'(x_0) = N_2(h) = 2N_1(\frac{h}{2}) - N_1(h) &= \frac{1}{h}[4f(x_0+\frac{h}{2}) - 3f(x_0) - f(x_0+h)] + \frac{h^2}{12}f'''(x_0) + \mathcal{O}(h^3) \\
N_2(\frac{h}{2}) = 2N_1(\frac{h}{4}) - N_1(\frac{h}{2}) &= \frac{1}{h}[8f(x_0+\frac{h}{4}) - 6f(x_0) - 2f(x_0+\frac{h}{2})] + \frac{h^2}{48}f'''(x_0) + \mathcal{O}(h^3)
\end{aligned}
$$

Then we eliminate the h^2 term and obtain the desired solution

$$
\begin{aligned}
f'(x_0) &= \frac{1}{3}[4N_2(\frac{h}{2}) - N_2(h)] = \frac{1}{h}[-7f(x_0) + \frac{32}{3}f(x_0+\frac{h}{4}) - 4f(x_0+\frac{h}{2}) + \frac{1}{3}f(x_0+h)] + \mathcal{O}(h^3) \\
\Longrightarrow f'(x_0) &= \frac{1}{3\cdot 4\cdot h}[-21f(x_0) + 32f(x_0+h) - 12f(x_0+2h) + f(x_0+4h)] + \mathcal{O}(h^3)\ .
\end{aligned}
$$

4.2.10

$$
\begin{aligned}
N_1(h) &= M - K_1h^2 - K_2h^4 - K_3h^6 + \cdots \\
N_1(\frac{h}{3}) &= M - K_1(\frac{h}{3})^2 - K_2(\frac{h}{3})^4 - K_3(\frac{h}{3})^6 + \cdots \\
N_1(\frac{h}{9}) &= M - K_1(\frac{h}{9})^2 - K_2(\frac{h}{9})^4 - K_3(\frac{h}{9})^6 + \cdots \\
\implies N_2(h) &= \frac{1}{8}[9N_1(\frac{h}{3}) - N_1(h)] = M + \frac{1}{9}K_2h^4 + \frac{10}{9^2}K_3h^6 + \cdots \\
N_2(\frac{h}{3}) &= \frac{1}{8}[9N_1(\frac{h}{9}) - N_1(\frac{h}{3})] = M + \frac{1}{9^3}K_2h^4 + \frac{10}{9^5}K_3h^6 + \cdots \\
\implies N_3(h) &= \frac{1}{80}[81N_2(\frac{h}{3}) - N_2(h)] = M - \frac{1}{9^3}K_3h^6 + \cdots = M + \mathcal{O}(h^6) \\
\implies N_3(h) &= \frac{1}{80}\left[81 \cdot \frac{1}{8}[9N_1(\frac{h}{9}) - N_1(\frac{h}{3})] - \frac{1}{8}[9N_1(\frac{h}{3}) - N_1(h)]\right] \\
&= \frac{1}{640}\left[N_1(h) - 90N_1(\frac{h}{3}) + 729N_1(\frac{h}{9})]\right] = M + \mathcal{O}(h^6)
\end{aligned}
$$